CORRECTION DES EXERCICES 18 A 41 P 153 SUR LES COMPLEXES
FORME TRIGONOMETRIQUE

Forme trigonométrique d'un nombre complexe

32 **1.** Les coordonnées polaires des points sont :

$A(4\,;0)$, $B\left(6\,;\frac{\pi}{4}\right)$, $C\left(3\,;\frac{\pi}{2}\right)$, $D\left(6\,;-\frac{3\pi}{4}\right)$, $E\left(6\,;-\frac{\pi}{2}\right)$, $F\left(6\,;\frac{5\pi}{6}\right)$ et $G\left(6\,;-\frac{\pi}{3}\right)$.

2. $\begin{cases}|z_1| = 5\\ \arg(z_1) = -\frac{\pi}{2}\ [2\pi]\end{cases}$ $\begin{cases}|z_2| = 3\\ \arg(z_2) = \pi\ [2\pi]\end{cases}$

$\begin{cases}|z_3| = \sqrt{3}+1\\ \arg(z_3) = \frac{\pi}{2}\ [2\pi]\end{cases}$ $\begin{cases}|z_4| = 2+\sqrt{2}\\ \arg(z_4) = -\frac{\pi}{2}\ [2\pi]\end{cases}$

33 • $|-1+i\sqrt{3}| = 2$, donc $-1+i\sqrt{3} = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$.

• $|1-i| = \sqrt{2}$, donc $1-i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$.

34 **a.** $\begin{cases}|z_1| = \sqrt{2}\times 2 = 2\sqrt{2}\\ \arg z_1 = \frac{3\pi}{4}+\frac{2\pi}{3} = \frac{17\pi}{12}\ [2\pi]\end{cases}$

b. $\begin{cases}|z_2| = 2\times 2\sqrt{2} = 4\sqrt{2}\\ \arg(z_2) = -\frac{\pi}{2}+\frac{\pi}{4} = -\frac{\pi}{4}\ [2\pi]\end{cases}$

c. $\begin{cases}|z_3| = \frac{\sqrt{3}+2}{\sqrt{8}} = \frac{\sqrt{3}+2}{2\sqrt{2}} = \frac{\sqrt{6}+2\sqrt{2}}{4}\\ \arg(z_3) = 0-\frac{\pi}{6} = -\frac{\pi}{6}\ [2\pi]\end{cases}$

d. $\begin{cases}|z_4| = \frac{\sqrt{12+36}}{1} = 4\sqrt{3}\\ \arg(z_4) = -\frac{\pi}{3}-\left(-\frac{\pi}{2}\right) = \frac{\pi}{6}\ [2\pi]\end{cases}$

35 **a.** $\begin{cases}|z_1| = 4\\ \arg(z_1) = \frac{\pi}{12}\ [2\pi]\end{cases}$ **b.** $\begin{cases}|z_2| = 2\\ \arg(z_2) = -\frac{\pi}{12}\ [2\pi]\end{cases}$

c. $\begin{cases}|z_3| = 3\\ \arg(z_3) = \pi-\frac{\pi}{12} = \frac{11\pi}{12}\ [2\pi]\end{cases}$

36 **1.** $|z| = 3\sqrt{2}$, $z = 3\sqrt{2}\left(-\frac{\sqrt{2}}{2} - i\,\frac{\sqrt{2}}{2}\right)$, $\arg(z) = \frac{5\pi}{4}\ [2\pi]$.

$|z'| = 2$, $z' = 2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\,i\right)$, $\arg(z') = \frac{\pi}{6}\ [2\pi]$.

2. $\begin{cases}|zz'| = 3\sqrt{2}\times 2 = 6\sqrt{2}\\ \arg(zz') = \arg(z)+\arg(z') = \frac{5\pi}{4}+\frac{\pi}{6} = \frac{17\pi}{12}\ [2\pi]\end{cases}$

$\begin{cases}\left|\frac{z}{z'}\right| = \frac{3\sqrt{2}}{2}\\ \arg\left(\frac{z}{z'}\right) = \arg(z)-\arg(z') = \frac{5\pi}{4}-\frac{\pi}{6} = \frac{13\pi}{12}\ [2\pi]\end{cases}$

37 $\begin{cases} |1+i| = \sqrt{2} \\ \arg(1+i) = \dfrac{\pi}{4} \quad [2\pi] \end{cases}$ $\begin{cases} |(1+i)^4| = |1+i|^4 = \sqrt{2}^4 = 4 \\ \arg(1+i)^4 = 4 \arg(1+i) = \pi \quad [2\pi] \end{cases}$

$$(1+i)^4 = 4(\cos\pi + i\sin\pi) = -4 .$$

39 **1.** $\begin{cases} |z_1| = 2 \\ \arg(z_1) = \dfrac{\pi}{3} \quad [2\pi] \end{cases}$ $\begin{cases} |z_2| = \sqrt{2} \\ \arg(z_2) = -\dfrac{\pi}{4} \quad [2\pi] \end{cases}$

$z_1 = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$ et $z_2 = \sqrt{2}\left(\cos\left(-\dfrac{\pi}{4}\right) + i\sin\left(-\dfrac{\pi}{4}\right)\right)$.

$|z_3| = \dfrac{|z_1|}{|z_2|} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$.

2. $\arg(z_3) = \arg(z_1) - \arg(z_2) = \dfrac{\pi}{3} + \dfrac{\pi}{4} = \dfrac{7\pi}{12} \quad [2\pi]$.

a. $z_3 = \sqrt{2}\left(\cos\dfrac{7\pi}{12} - i\sin\dfrac{7\pi}{12}\right)$.

b. $z_3 = \dfrac{1+i\sqrt{3}}{1-i} = -\dfrac{\sqrt{3}}{2} + \dfrac{1}{2} + i\left(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\right)$.

$\begin{cases} \sqrt{2}\cos\dfrac{7\pi}{12} = \dfrac{-\sqrt{3}+1}{2} \\ \sqrt{2}\sin\dfrac{7\pi}{12} = \dfrac{\sqrt{3}+1}{2} \end{cases}$ $\begin{cases} \cos\dfrac{7\pi}{12} = \dfrac{-\sqrt{6}+\sqrt{2}}{4} \\ \sin\dfrac{7\pi}{12} = \dfrac{\sqrt{6}+\sqrt{2}}{4} \end{cases}$

40 **1.** $z_1 = \sqrt{2}\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i\right) = \sqrt{2}\left[\cos\left(-\dfrac{\pi}{3}\right) + i\sin\left(-\dfrac{\pi}{3}\right)\right]$.

$z_2 = \sqrt{2}\left(\dfrac{\sqrt{2}}{2} - i\dfrac{\sqrt{2}}{2}\right) = \sqrt{2}\left[\cos\left(-\dfrac{\pi}{4}\right) + \sin\left(-\dfrac{\pi}{4}\right)\right]$.

$Z = \dfrac{z_1}{z_2} = 1\left[\cos\left(-\dfrac{\pi}{3} + \dfrac{\pi}{4}\right) + i\sin\left(-\dfrac{\pi}{3} + \dfrac{\pi}{4}\right)\right]$.

$Z = \cos\left(-\dfrac{\pi}{12}\right) + i\sin\left(-\dfrac{\pi}{12}\right)$.

2. $Z = \dfrac{z_1}{z_2} = \dfrac{1+i\sqrt{z}}{1+i} = \dfrac{\sqrt{6}+\sqrt{2}}{4} + i\left(\dfrac{\sqrt{6}-\sqrt{2}}{4}\right)$.

En identifiant $\cos\left(-\dfrac{\pi}{12}\right) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ et $\sin\left(-\dfrac{\pi}{12}\right) = \dfrac{\sqrt{6}-\sqrt{2}}{4}$,

on a : $\cos\dfrac{\pi}{12} = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ et $\sin\dfrac{\pi}{12} = -\dfrac{\sqrt{6}-\sqrt{2}}{4}$.

41 • $z^2 = 25(-\sqrt{2+\sqrt{2}} + i\sqrt{2-\sqrt{2}})^2$

$= 25(2+\sqrt{2} - 2i\sqrt{(2+\sqrt{2})(2-\sqrt{2})} - 2 + \sqrt{2})$

$= 100\left(\dfrac{\sqrt{2}}{2} - i\dfrac{\sqrt{2}}{2}\right) = 100\left[\cos\left(-\dfrac{\pi}{4}\right) + i\sin\left(-\dfrac{\pi}{4}\right)\right]$.

$\begin{cases} |z|^2 = 100 \\ \arg(z^2) = -\dfrac{\pi}{4} \quad [2\pi] \end{cases}$ $\begin{cases} |z| = 10 \\ 2\arg(z) = -\dfrac{\pi}{4} \quad [2\pi] \end{cases}$ $\begin{cases} |z| = 10 \\ \arg(z) = -\dfrac{\pi}{8} \quad [\pi] \end{cases}$

• Il y a deux arguments possibles : $-\dfrac{\pi}{8}$ ou $\dfrac{7\pi}{8}$, or :

$\mathrm{Re}(z) < 0$ et $\mathrm{Im}(z) > 0$; donc :

$\arg(z) \in \left[\dfrac{\pi}{2} ; \pi\right]$, d'où $\arg(z) = \dfrac{7\pi}{8} \quad [2\pi]$.