CORRECTION DES EXERCICES SUR LE CALCUL INTEGRAL P 233 EX 56 A 62

Corrigés des ex 56 à 62

56. $I = \int_0^{\pi} \cos 3x \sin^4(3x)\,dx$ Rq : on reconnaît $\int u'u^4$ à une constante près

$I = \int_0^{\pi} \frac{1}{3} \cdot 3 \cos 3x \sin^4(3x)\,dx$

$I = \frac{1}{3} \int_0^{\pi} 3 \cos 3x \sin^4 3x\,dx = \frac{1}{3}\left[\frac{\sin^5 3x}{5}\right]_0^{\pi}$

$I = 0.$

$J = \int_0^{\frac{\pi}{2}} \sin 2x\,(1-\cos 2x)^5\,dx$

$J = \frac{1}{2} \int_0^{\frac{\pi}{2}} 2 \sin 2x\,(1-\cos 2x)^5\,dx = \frac{1}{2}\left[\frac{(1-\cos 2x)^6}{6}\right]_0^{\frac{\pi}{2}}$

$J = \frac{1}{12}\left[(1+1)^6 - (1-1)^6\right] = \frac{1}{12} \cdot 2^6 = \frac{2^4}{3} = \frac{16}{3}$

57. a. $\int_0^1 \frac{t^3\,dt}{\sqrt{t^4+1}} = \frac{1}{4}\int_0^1 \frac{4t^3}{\sqrt{t^4+1}}\,dt = \frac{1}{2}\left[\sqrt{t^4+1}\right]_0^1 = \frac{\sqrt{2}-1}{2}$

b. $I = \int_0^1 \frac{u^3+1}{u^4+4u+2}\,du = \frac{1}{4}\int_0^1 \frac{4u^3+4}{u^4+4u+2}\,du = \frac{1}{4}\left[\ln|u^4+4u+2|\right]_0^1$

$I = \frac{1}{4} \ln \frac{7}{2}$

c. $\int_0^1 e^{3x-1}\,dx = \frac{1}{3}\int_0^1 3e^{3x-1}\,dx = \frac{1}{3}\left[e^{3x-1}\right]_0^1 = \frac{1}{3}\left(e^2 - \frac{1}{e}\right)$

d. $\int_0^1 t e^{t^2}\,dt = \frac{1}{2}\int_0^1 2t e^{t^2}\,dt = \frac{1}{2}\left[e^{t^2}\right]_0^1 = \frac{1}{2}(e-1)$

58 - a $I = \int_{-1}^{0} \frac{1}{2t-1}\,dt = \frac{1}{2}\int_{-1}^{0} \frac{2}{2t-1}\,dt = \frac{1}{2}\left[\ln|2t-1|\right]_{-1}^{0} = \frac{1}{2}\ln\frac{1}{3}.$

Attention ici $2t-1<0$ car t ds $[-1;0]$

b- $J = \int_0^{\ln 2} e^{-x+1}\, dt = -\int_0^{\ln 2} -e^{-x+1}\, dx = -\left[e^{-x+1} \right]_0^{\ln 2}$

$J = e - e^{-\ln 2 + 1} = e - e^1 \times e^{\ln \frac{1}{2}} = e - \frac{1}{2} e = \frac{1}{2} e.$

69- a- $I = \int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx = \left[\ln (e^x + e^{-x}) \right]_0^1 = \ln\left(e + \frac{1}{e}\right) - \ln 2$

$I = \ln \left[\frac{e^2+1}{2e} \right].$

b- $J = \int_0^1 \sqrt{3t+1}\, dt = \frac{1}{3} \int_0^1 3\sqrt{3t+1}\, dt = \frac{1}{3} \int_0^1 3(3t+1)^{\frac{1}{2}}\, dt$

$J = \frac{1}{3} \left[\frac{2(3t+1)^{\frac{3}{2}}}{3} \right]_0^1 = \frac{2}{9} \left[\sqrt{(3t+1)^3} \right]_0^1$

$J = \frac{14}{9}.$

c- $\int_{-1}^1 \frac{x-1}{(x^2-2x+5)^4}\, dx = \frac{1}{2} \left[-\frac{1}{3} \cdot \frac{1}{(x^2-2x+5)^3} \right]_{-1}^1 = \frac{-7}{3072}$

60- $I = 3\int_3^4 \frac{2x-1}{(x^2-x)^3}\, dx = 3 \left[-\frac{1}{2} \cdot \frac{1}{(x^2-x)^2} \right]_3^4 = \frac{1}{32}$

$J = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{\pi}{2} - 2x\right) \cos x\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(-\frac{\pi}{2} + 2x\right) \cos x\, dx$

car $\left| \frac{\pi}{2} - 2x \right| = \begin{cases} \frac{\pi}{2} - 2x & \text{si } x \leq \frac{\pi}{4} \\ -\frac{\pi}{2} + 2x & \text{si } x \geq \frac{\pi}{4} \end{cases}$

donc $J = \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\pi}{2} \cos x\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2x \cos x\, dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\pi}{2} \cos x\, dx$

car $[x \mapsto 2x \cos x]$ est une fonction impaire.

$$J = \frac{\pi}{2}\left[\sin x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{2}} + 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos x\,dx - \frac{\pi}{2}\left[\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$J = \frac{\pi}{2}\left[1 + \frac{\sqrt{2}}{2}\right] + 2R - \frac{\pi}{2}\left[1 - \frac{\sqrt{2}}{2}\right]$$

$$J = \frac{\pi\sqrt{2}}{2} + 2R$$

$$R = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos x\,dx = \left[x\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x\,dx$$

$$R = \frac{\pi}{2} - \frac{\pi\sqrt{2}}{8} + \left[\cos x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{\pi}{2} - \frac{\pi\sqrt{2}}{8} + 0 - \frac{\sqrt{2}}{2}$$

$$2R = \pi - \frac{\pi\sqrt{2}}{4} - \sqrt{2}$$

$$J = \frac{\pi\sqrt{2}}{4} + \pi - \sqrt{2}.$$

61 – $I = \int_0^{\frac{\pi}{3}} \frac{x}{\cos^2 x}\,dx = \left[x\tan x\right]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \tan x\,dx$

$v(x) = x \qquad v'(x) = 1 \qquad I = \frac{\pi}{3}\tan\frac{\pi}{3} + \left[\ln|\cos x|\right]_0^{\frac{\pi}{3}}$

$u'(x) = \frac{1}{\cos^2 x} \qquad u(x) = \tan x = \frac{\sin x}{\cos x}$

$$I = \frac{\pi}{3}\sqrt{3} + \ln\frac{1}{2} = \frac{\pi\sqrt{3}}{3} - \ln 2.$$

$$J = \int_0^{\frac{\pi}{2}} x\cos^2 x + \sin^4 x\,dx = \int_0^{\frac{\pi}{2}} x\cos^2 x\,dx + \int_0^{\frac{\pi}{2}} \sin^4 x\,dx$$

On linéarise $\sin^4 x = \frac{1}{8}\left[\cos 4x - 4\cos 2x + 3\right]$ et $\cos^2 x = \frac{1+\cos 2x}{2}$

$$\int_0^{\frac{\pi}{2}} x\cos^2 x\,dx = \int_0^{\frac{\pi}{2}} \frac{x}{2}\,dx + \frac{1}{2}\int_0^{\frac{\pi}{2}} x\cos 2x\,dx$$

$$= \left[\frac{x}{2}\sin 2x\right]_0^{\frac{\pi}{2}} - \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 2x\,dx$$

Très difficile pour rien ! $J = \frac{1}{16}\pi^2 - \frac{1}{4} + \frac{3}{16}\pi$

des erreurs ont dû se cacher ds l'énoncé !

62 - a- $I = \frac{1}{3}\left[\frac{\sin^5 3x}{5}\right]_0^{\pi} = 0$

b- $\int_1^2 |2x+3|\, dx = \int_1^2 2x+3\, dx = \left[x^2+3x\right]_1^2 = 6$

c- $\int_0^2 \left[1-(x-1)\right]^3 dx = \int_0^2 x^3\, dx = \left[\frac{x^4}{4}\right]_0^2 = 4.$

d- $\int_{-6}^4 |x^2-2x-3|\, dx = \int_{-6}^{-1} x^2-2x-3\, dx + \int_{-1}^3 -x^2+2x+3\, dx$

$+ \int_3^4 x^2-2x-3\, dx = \frac{314}{3}$

$x^2-2x-3 \geq 0$ si $x \in \left]-\infty ; -1\right] \cup \left[3 ; +\infty\right[.$